Introduction to *$0\nu\beta\beta$ Theory*

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The Majorana vs. Dirac Question

Is each neutrino mass eigenstate, such as ν_1,

a Majorana fermion $\overline{\nu_1} = \nu_1$

or

a Dirac fermion $\overline{\nu_1} \neq \nu_1$

If there are ***Majorana mass terms***,
the neutrino mass eigenstates
will be Majorana particles.

Majorana Mass Terms

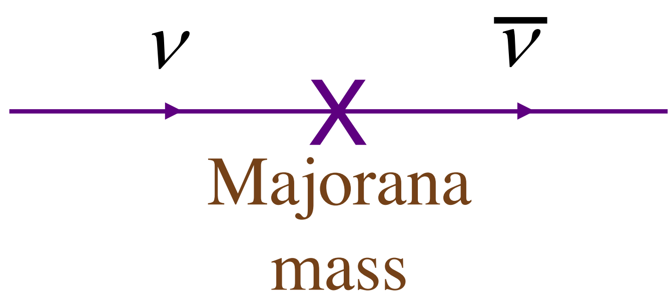
Suppose ν_L and ν_R are underlying chiral fields in terms of which the SM, extended to include neutrino mass, is written. The antineutrinos these fields create are distinct from the neutrinos they absorb.

In terms of these underlying fields, a Majorana mass term has the form —

Charge conjugate

$$\overline{(\nu_L)^c}\,\nu_L + h.c. \quad \text{or} \quad \overline{(\nu_R)^c}\,\nu_R + h.c.$$

Either of these has the effect —

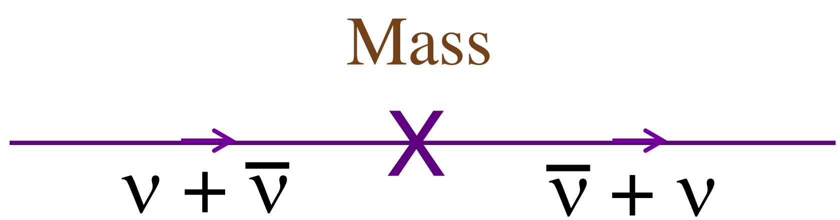


and

Then the mass eigenstate neutrino "ν_1" must be —

$$\nu_1 = \nu + \bar{\nu} \ ,$$

since this is the neutrino that the Majorana mass term sends back into itself, as required for any mass eigenstate particle:



When the underlying mass is a Majorana mass, the mass eigenstate will be a Majorana neutrino.

If the masses of the neutrino mass eigenstates come from ***both*** Majorana mass terms and Standard Model-style "Dirac" mass terms, the neutrino mass eigenstates will still be Majorana particles.

Reasons To Believe Neutrinos May Well Be Majorana Particles

"What other kind of neutrinos is there?"

Lincoln Wolfenstein

The most popular theory of why neutrinos are so light is the See-Saw model.

Straightforward See-Saw models predict that neutrinos are Majorana particles.

To Determine Whether Majorana Masses Occur in Nature, So That $\bar{\nu} = \nu$

The Promising Approach — Seek

Neutrinoless Double Beta Decay [$0\nu\beta\beta$]

Observation at any non-zero level would imply —

- Lepton number L is not conserved ($\Delta L = 2$)
- Neutrinos have Majorana masses
- Neutrinos are Majorana particles (self-conjugate)

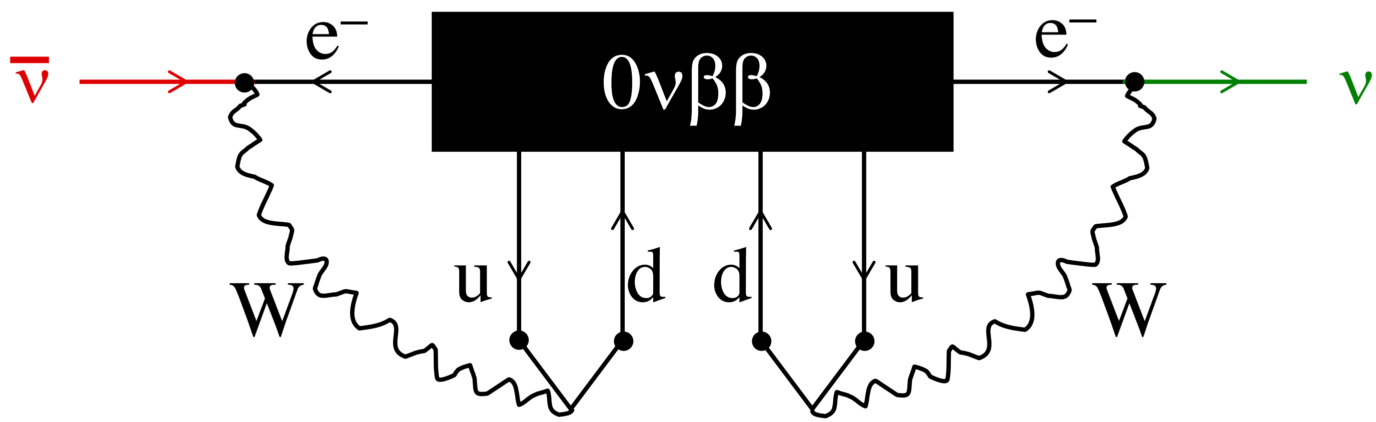
$0\nu\beta\beta$ and the Matter-Antimatter Asymmetry of the Universe

Leptogenesis explains the present-day matter-antimatter asymmetry in terms of early-universe decay processes that do not conserve the lepton number L.

Observation of $0\nu\beta\beta$ would establish that indeed L is not always conserved, making Leptogenesis more likely.

Whatever diagrams cause $0\nu\beta\beta$, its observation would imply the existence of a Majorana mass term:

(Schechter and Valle)

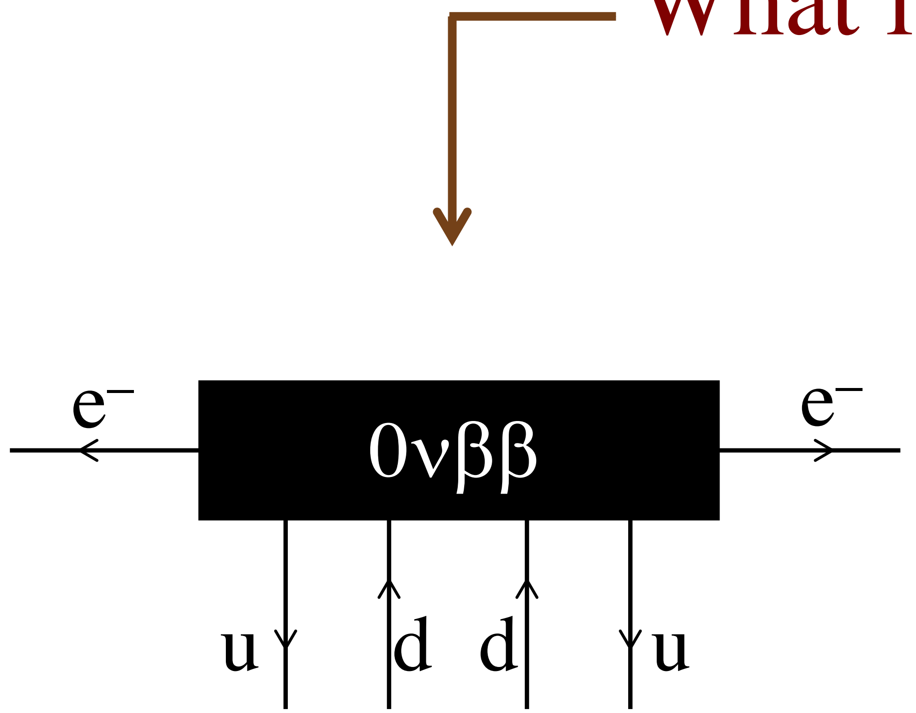


$\bar{\nu} \rightarrow \nu$: A (tiny) Majorana mass term

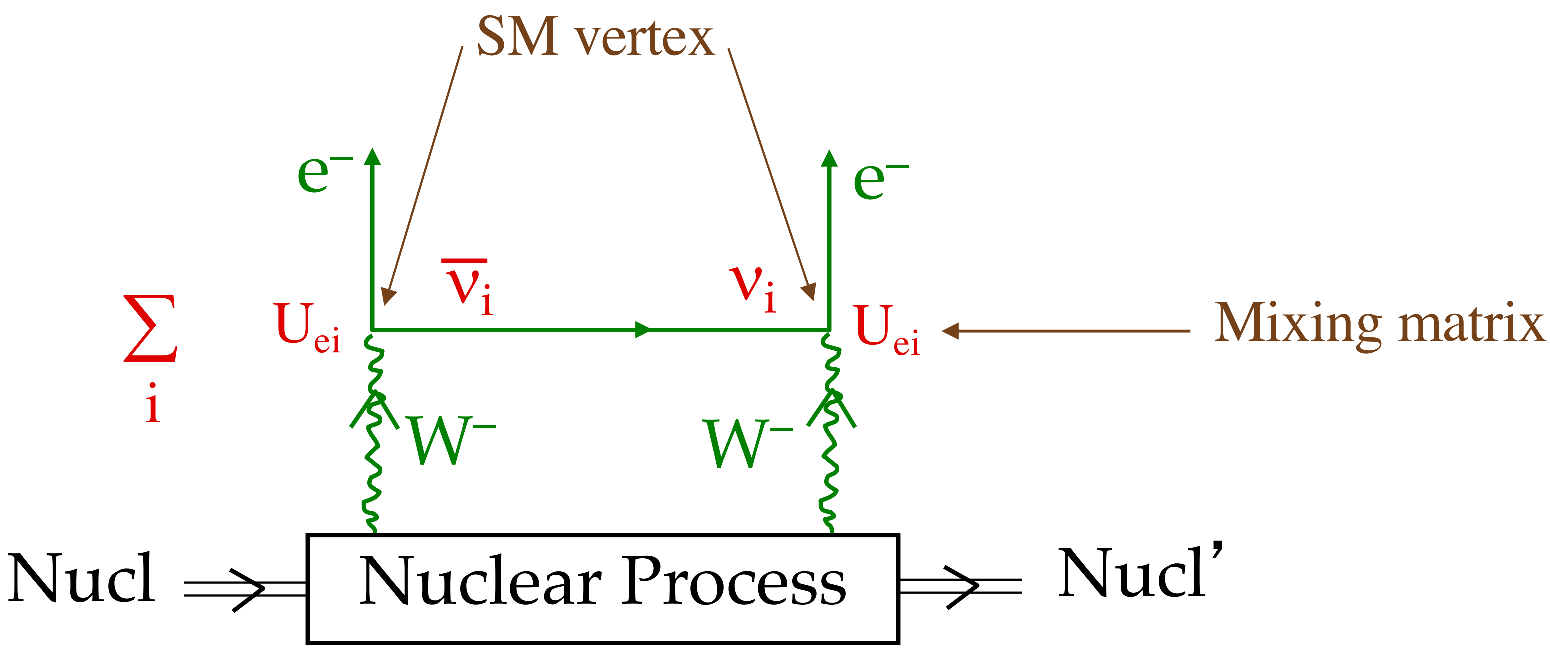
(Duerr, Lindner, Merle)

$\therefore\ 0\nu\beta\beta \Longrightarrow \bar{\nu}_i = \nu_i$

What is inside?



We anticipate that $0\nu\beta\beta$ is dominated by a diagram with light neutrino exchange and Standard Model vertices:

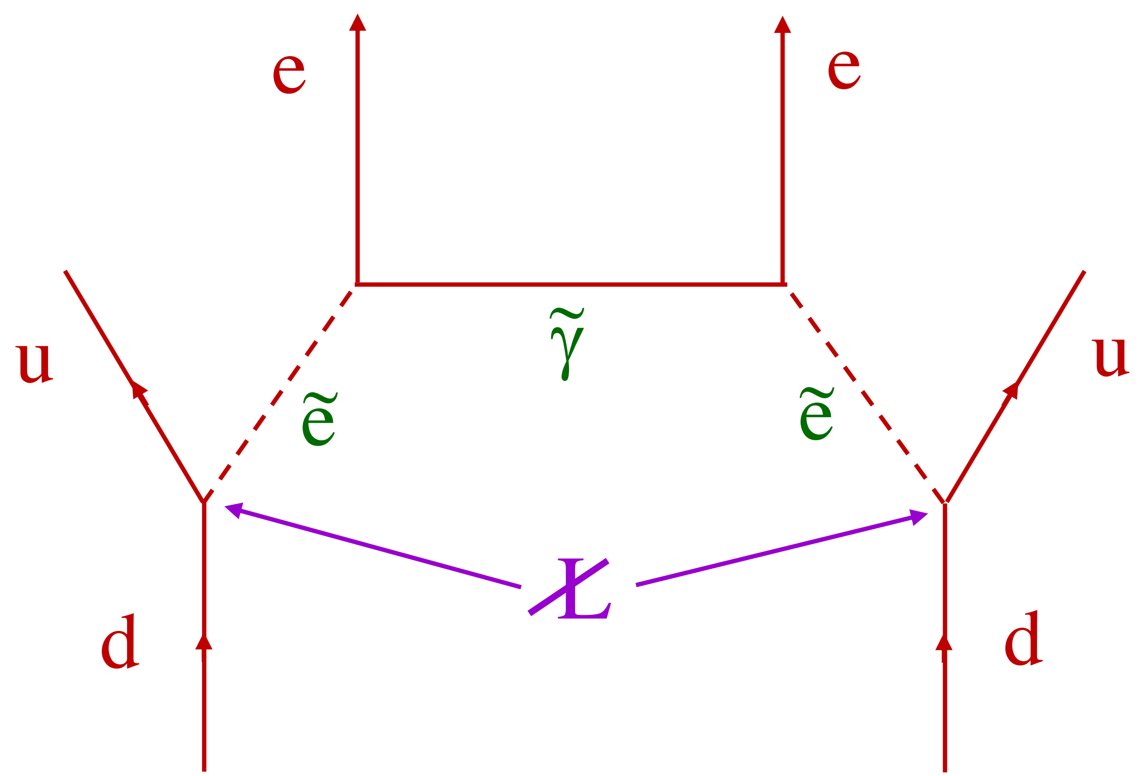


"The Standard Mechanism"

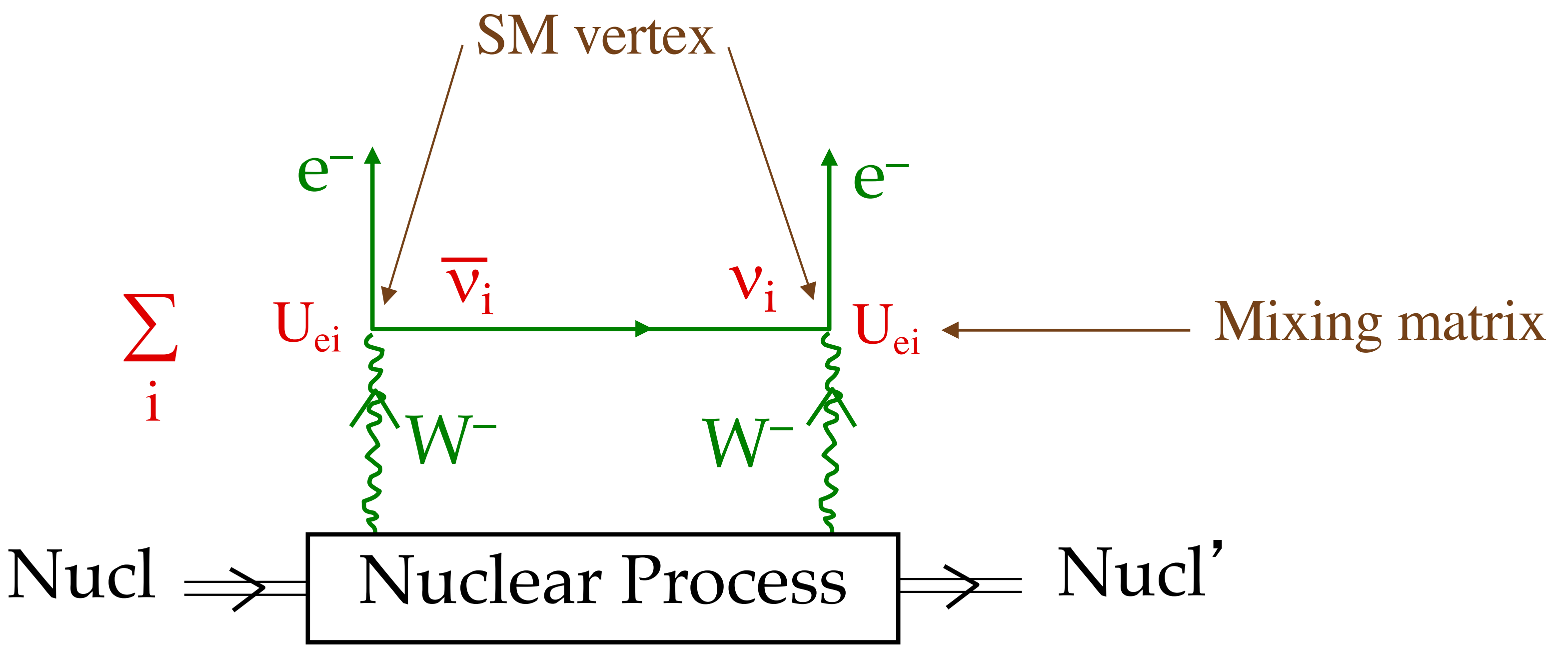
Of ourse, there could be other contributions to $0\nu\beta\beta$, which at the quark level is the process

$dd \rightarrow uuee$.

An example from Supersymmetry:



If the dominant mechanism is –



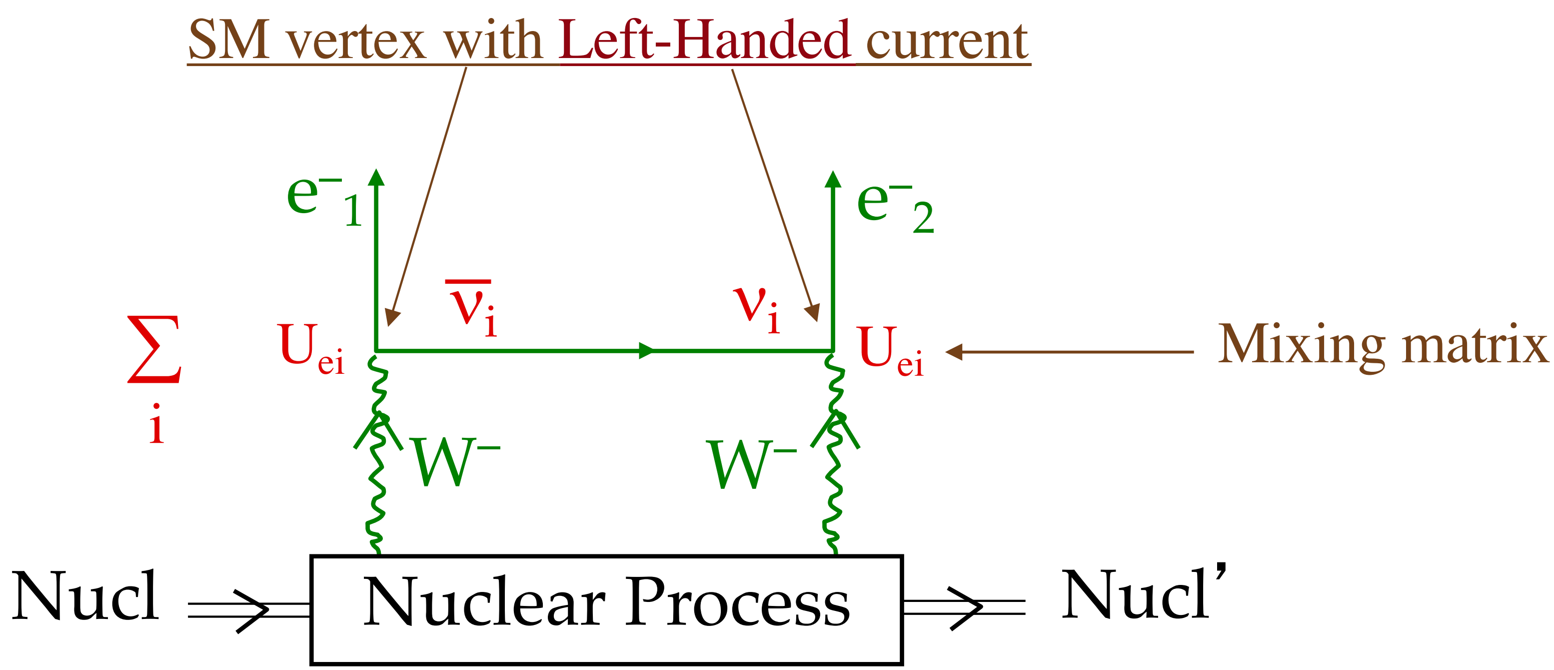
Then –

Mass (ν_i)

$$\mathrm{Amp}[0\nu\beta\beta] \propto \left| \sum m_i U_{ei}^{\;2} \right| \equiv m_{\beta\beta}$$

The proportionality involves a nuclear matrix element that is uncertain by a factor of 2 or 3.

Why Is $m_{\beta\beta} \equiv \left|\sum m_i U_{ei}^2\right|$ Proportional To Neutrino Masses m_i?



The amplitude contributed by ν_i exchange then involves the factor –

$$\underbrace{U_{ei}\bar{u}_1\gamma_\mu P_L}_{\text{Vertex}}\overbrace{\frac{\left(\not{q}+m_i\right)}{q^2-m_i^2}}^{\nu_i\text{ Propagator}}\underbrace{P_L\gamma_\nu v_2 U_{ei}}_{\text{Vertex}}$$

Here, $P_L \equiv \left(1-\gamma_5\right)/2$ is the Left-Handed chirality projection operator.

$P_L\not{q}P_L = P_L P_R\not{q} = 0$, so the $\not{q}$ term does not contribute.

P_R: $\left(1+\gamma_5\right)/2$, the Right-Handed chirality projection operator.

Only the m_i term contributes.

What if the current at one vertex is a non-SM Right-Handed current?

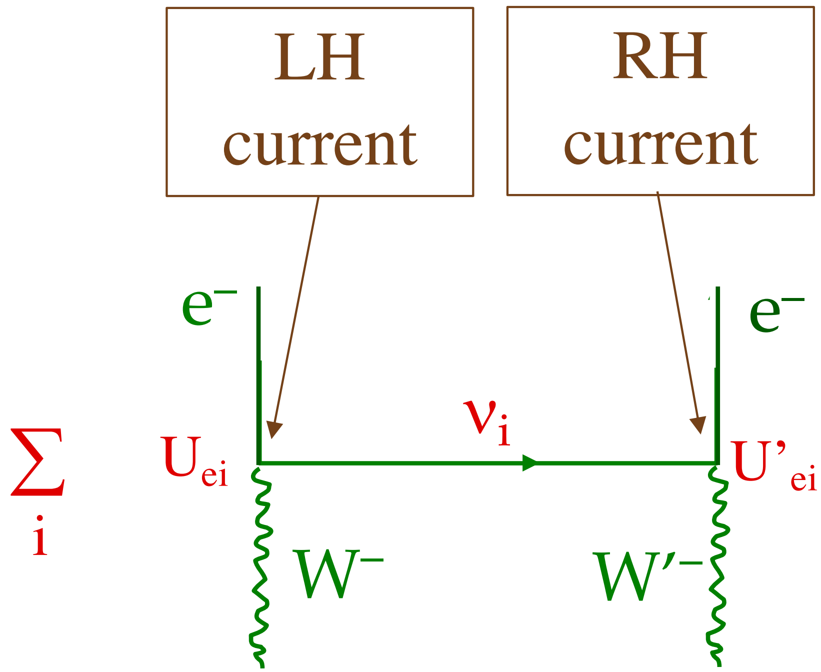
We then have –

$$U_{ei}\bar{u}_1\gamma_\mu P_L \frac{(\not{q}+m_i)}{q^2-m_i^2} P_R \gamma_\nu v_2 U'_{ei}$$

$P_L \not{q} P_R = P_L P_L \not{q} = P_L \not{q}$, so now the $\not{q}$ term does contribute.

The ν_i contribution is no longer proportional to its mass.

However, in —



any individual ν_i term violates unitarity when the energy is large. Unitarity violation by the sum can then be avoided only through a cancellation between the terms.

Thus, we require that

$$\sum_i U_{ei} \frac{\not{q}}{q^2 - m_i^2} U'_{ei}$$

vanish when q^2 is large.

This requires that $\sum_i U_{ei} U'_{ei} = 0$.

But then, if all $m_i = 0$, the amplitude for *0νββ* with a LH current at one vertex and a RH current at the other vanishes completely.

0νββ requires non-zero neutrino masses, even when there is a RH current at one of its two vertices.

Petcov

The Possible Size of $m_{\beta\beta} = \left|\sum_i m_i U_{ei}^2\right|$

The leptonic mixing matrix U is —

$c_{ij} \equiv \cos\theta_{ij}$
$s_{ij} \equiv \sin\theta_{ij}$

$$
\begin{array}{cccc}
 & \nu_1 & \nu_2 & \nu_3 \\
U = & \multicolumn{3}{l}{\left[\begin{array}{ccc}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{array}\right]}
\end{array}
$$

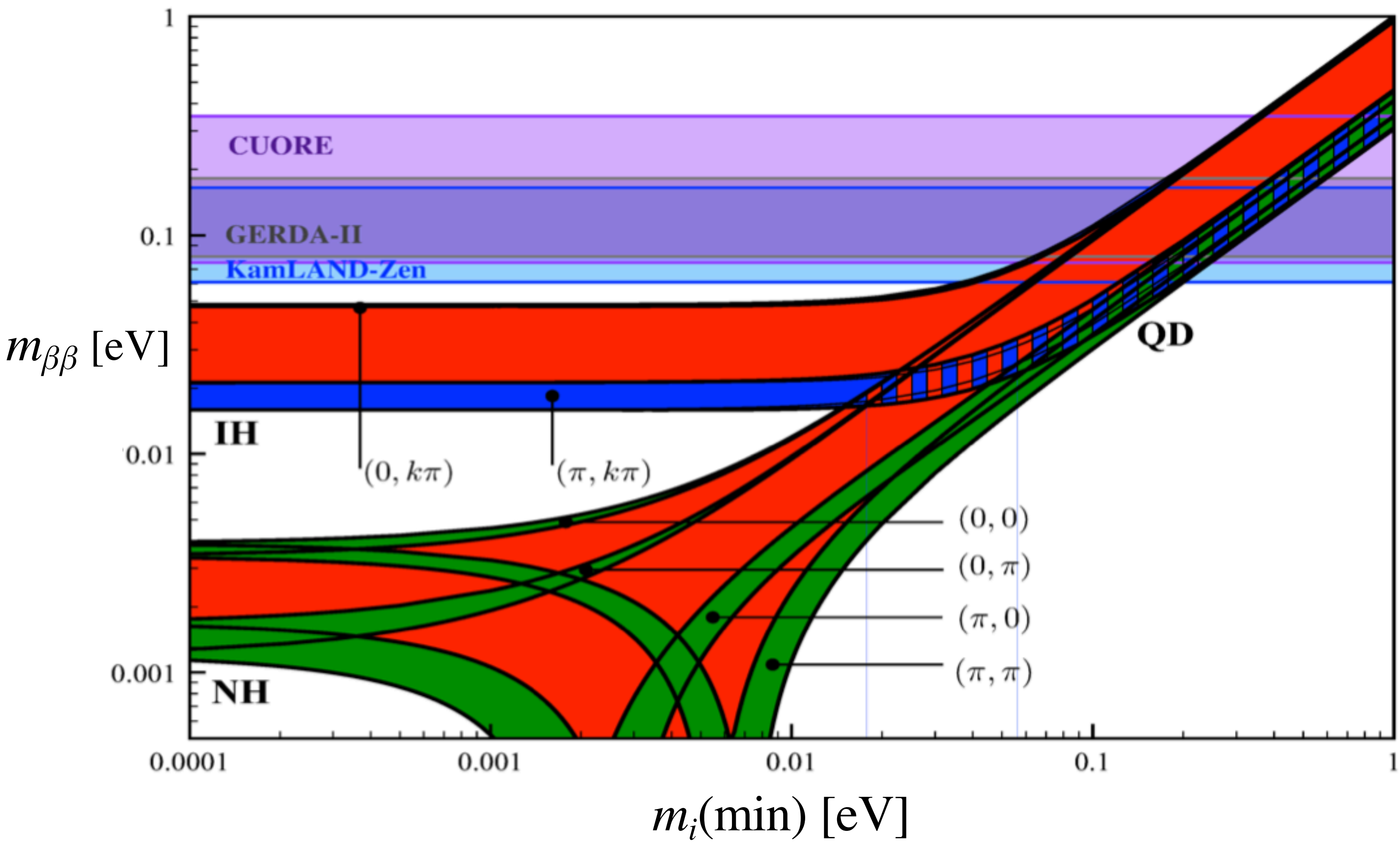
$$\times \mathrm{diag}\left(\underbrace{e^{i\alpha_1/2}, e^{i\alpha_2/2}}_{\text{Majorana phases}}, 1\right)$$

Majorana phase α_i is associated with neutrino ν_i.

Majorana phases ⟵ Affect only $\cancel{L}$ processes

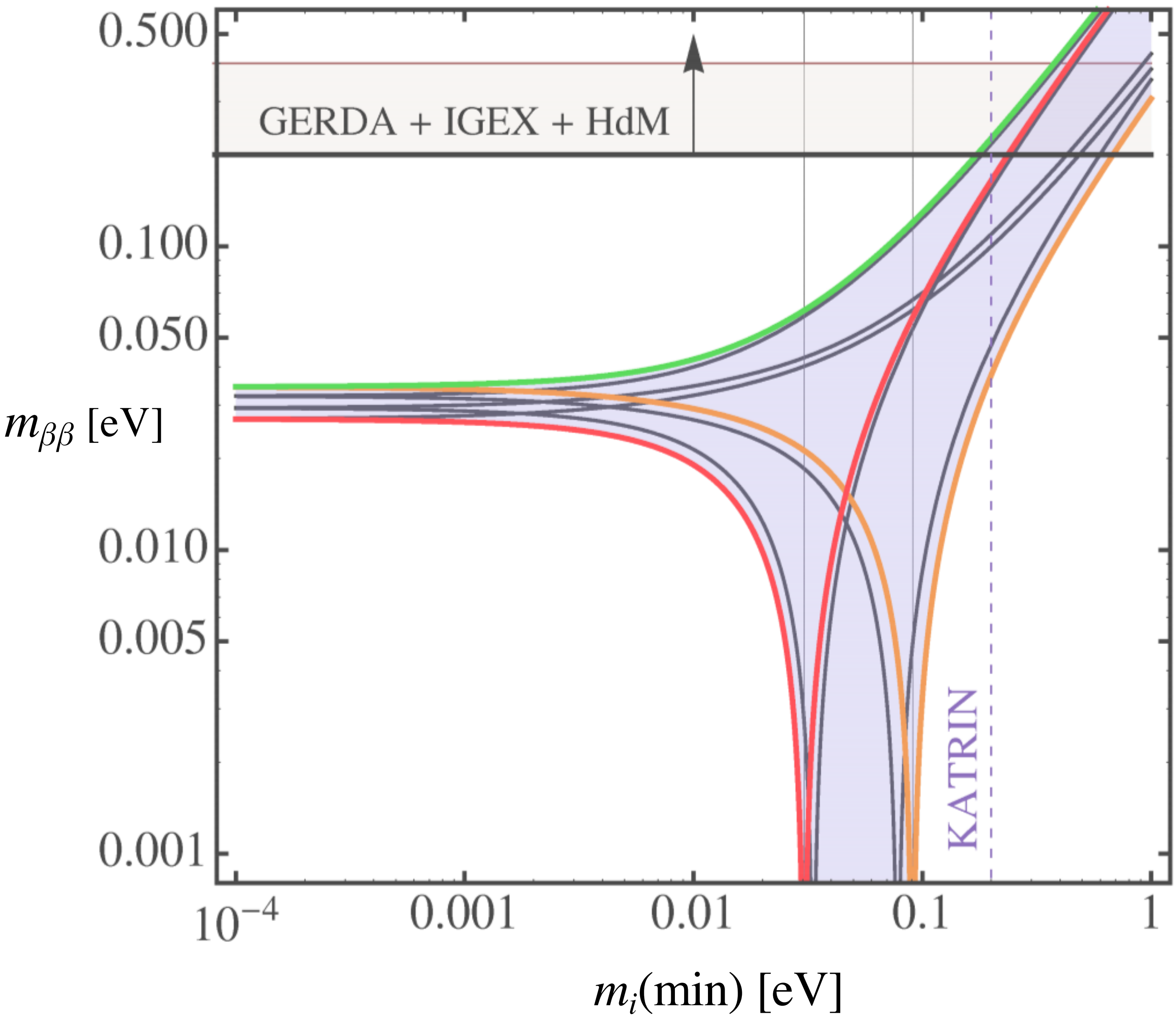
$$\text{Thus } m_{\beta\beta} = \left| m_1 c_{12}^2 c_{13}^2 e^{i\alpha_1} + m_2 s_{12}^2 c_{13}^2 e^{i\alpha_2} + m_3 s_{13}^2 e^{-2i\delta} \right| .$$

Possible Size of $m_{\beta\beta}$



Pascoli

If there is a 4^{th} neutrino mass eigenstate, the picture changes. Example:



NH

$$\Delta m^2_{41} \cong 1.8\,\mathrm{eV}^2$$

$$|U_{e4}|^2 \cong 0.02$$

Girardi et al.

Closing Comment

We all want to know whether neutrinos are Majorana particles.

The observation of $0\nu\beta\beta$ would establish that indeed they are.

I am sure that most of us hope that $0\nu\beta\beta$ will be observed soon.